Homework 2: Truncation error and machine precision problems

due date: October 17 2018

You will need to review plotting and function handles for this homework.

1. Show the effect of the truncation error by comparing the functions, relative error, and absolute error. Use this function

$$f(x) = exp(x)$$

Construct an approximation to the function using the Taylor series.

   (a) (10 points) write and document a function `approxexp(x,n)` where n is the number of elements in the series, the function should return a vector of the same size as x. [name the file approxexp.m]

   (b) (10 points) write and document a function `abserror(x,@f1,@f2)` where $@f1$ is the function handle to the approximation and $@f2$ is function handle for the comparison function, for our example would call our function `abserror(x,@approxexp,@yourexp)`. The function will return a vector of the same size as x.

   (c) (10 points) write and document a function `relerror(@f1,@f2)` where $@f1$ is the function handle to the approximation and $@f2$ is function handle for the comparison function, for our example would call our function `abserror(@approxexp,@yourexp)`. The function will return a vector of the same size as x.

   (d) (10 points) create a file homework2.m that contains the code for all plots of this homework

   - plot the function over the domain of -3 to 3 and approximations with $n = 3, n = 4$, and $n = 5$ elements from the Taylor series using your `approxexp(x,n)` into a single plot.
   - plot the absolute errors for the $approxexp(x, n)$ for the three n.
   - plot the relative errors

2. (20 points) Explore the effect of the evaluation of the central difference for the function $g(x)$ for various stepsizes h, discuss stepsize h and back up your discussion with

plots (use the absolute error for each value for $x = 0 : 0.1 : 1$ for different h). In the plot, I expect to see problematic h and your finding what the best choice of h should be. I do not expect an formal numerical analysis but comparisons of stepsizes such as $h = 10^{-10}$,$h = 10^{-16}$, or $h = 10^{-5}$ among others, decide yourself which you want to show, but plot several (more than 2 but less than 10) into one plot, and use a legend to tell us about the size of h. And discuss the result, I expect a paragraph.

The function

$$g(x) = 1 - e^{-x\lambda}$$

$g(x)$ is the CDF of the exponential distribution (so the central difference $\frac{g(x+h)-g(x-h)}{2h}$ approximates the PDF of the Exponential distribution (which is $g'(x) = \lambda e^{-x\lambda}$. Use $\lambda = 1$. Add this second part also to the homework2.m

Submit a PDF of homework2.m, make sure that the document is self contained and has neough text we can read and understand what is going on, submit the other .m as a zip file also to canvas.